CONTINUOUS-TIME STOCHASTIC PROCESSES

Module 3, 2021–2022

Professor: Andrei Savochkin

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Course information

Course Website: my.nes.ru

Instructor's Office Hours: TBD

Class Time: TDB

Room: TBD

TA: TBD

Course description and prerequisites

This course teaches the basic ideas of the theory of continuous-time stochastic processes and stochastic calculus. The goal is to enable students to understand models in Economics and Finance that rely on continuous-times stochastic processes as modeling devices. The target audience of the course is students who did not have any formal training in stochastic processes as undergraduates but wish to expand the range of tools and methods that they are familiar with. The prerequisites for the course are basic knowledge of probability theory and good command of mathematical analysis; knowledge of the theory of measure and integration is a big plus but is not strictly necessary.

Due to prerequisites and limited class time, the course is not designed to be 100% formal and rigorous. We will be doing some proofs to have a better understanding of what claims are true and what are false and master the studied concepts. However, many more demanding proofs (e.g., existence theorems) will be omitted. Three lectures are planned to be spent on applications.

Course requirements, grading, and attendance policies

Student's achievements will be evaluated on the basis of problem sets, the final exam, and class participation.

The total score is computed on the 100-point scale as a weighted sum of problem set scores and the exam score and then converted to transcript grades (from 2 to 5+). There will be four problem sets (possibly, of unequal size and weight) in which the sum of points for all problems will be around 100 points. The total weight of problem sets in the final grade is 24%.

The final exam is graded on the scale of 0 to 100 points and enters the total score with the 76% weight. The format of the exam is "A4." Each student is allowed to bring to the exam one sheet of paper of A4 size (double-sided) with notes, handwritten or typed.

For inclusion into the total score, the exam score is adjusted for class participation. In the online teaching format, class participation requires presence with video turned on and participation in polls. During each lecture, there is one or more polls that require students quickly express their opinion on some question (e.g., "Is this a correct solution of the equation, yes or no?"). If a student misses a substantial number of polls by not answering the question, then he or she is regarded as not participating. (Exceptions may be granted by the academic office in case of illness or the instructor in extraordinary circumstances by advance agreement.) For students who miss four or fewer meetings, the adjusted score equals to the raw exam score. For students who miss 5 meetings, the adjusted score is the raw score minus 5 points. Students who miss 6 meetings get a 12 point deduction, students who miss 7 meetings get a 20 point deduction, and each subsequent missed meeting incurs an additional 8 point deduction.

Course contents

The course covers the following topics.

1. Key concepts and theorems from the probability theory and the theory of integrals
   - σ-fields, measures, convergence, integrals, stochastic processes, filtrations, conditional expectations
2. Continuous-time martingales
   - the concept, stopping time, optional sampling theorems
3. Brownian motion
   - definition and properties, the Ito and Stratonovich integrals, Ito's rule
4. Martingales and stochastic integral
   - the martingale representation theorem, Levy's characterization of the Brownian motion, change of probability measure and the Girsanov theorem
5. Introduction to stochastic differential equations
6. Applications
   - Asset Pricing, Principal-Agent problems

Description of course methodology

Most of course material will be presented in lectures and sections meetings. Due to the complexity of the studied concepts, it is also recommended to review lecture notes and (read the textbook if needed) before each class.

Sample task for course evaluation

(Oksendal, Ex. 4.5) Let B_t be a one-dimensional Brownian motion with $B_0 = 0$. Define $\beta_k(t) = E[B_t^k]$ for $k \in \mathbb{Z}$.

Prove that $\beta_k(t) = \frac{1}{2}k(k-1)\int_0^t \beta_{k-2}(s)\,ds$ for all $k \geq 2$.

Course materials

Required textbooks and materials

The required textbook is

- Oksendal B., "Stochastic Differential Equations: An introduction with Applications," 6th edition.

Besides that, we will study a few papers that will be announced later.

Additional materials

A much more detailed and rigorous textbook is

- Karatzas, I., and S. E. Shreve, "Brownian Motion and Stochastic Calculus," 2nd edition.

For the primer on measure and integral, recommended textbooks are

- Athreya, K. B., and S. N. Lahiri, "Measure Theory and Probability Theory,"
- Royden, H. L., "Real Analysis," 3rd (not 4th!) edition.

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.